Mathematical Methods in Economics

The objective of this course is to endow first year masters students with the requisite tools needed in advanced courses on microeconomics, macroeconomics and econometrics. Since this course will be taught concurrently with Microeconomics I and Macroeconomics I, an introductory two-week (12 hours) bridge course is intended to be offered. This would essentially provide students with a review of some of the mathematical techniques described below.

Section I
Review of Basic Mathematical Methods

1) Set Theory and the Real Number System:
   (a) Elementary Sentential and Predicate Logic, Vocabulary of Sets, Set Operations
   (b) Field and Order Axioms, Natural Numbers, Integers and Rationals, Real Numbers
   (c) Proof Construction: Direct and Indirect Proofs, Converse and Contrapositive, Mathematical Induction.

2) One Variable Calculus: Introduction
   (a) Functions and Relations: Relations, Functions, Inverse and Composition of Functions, Monotonic Functions.
   (b) Limits, Continuity and Differentiability: The Limits of a Function, Continuous Functions, The Derivative, Chain Rule, The Mean Value Theorem, Partial Derivative, Total Derivative.
   (c) Integral Calculus, Integration By Parts, The Fundamental Theorem of Calculus.
   (d) Convex and Concave Functions
   (e) Graphing Functions: Using derivatives for drawing graphs of functions.

3) One Variable Calculus: Optimization
   (a) Unconstrained Maximization: Local and Global Maxima, First and Second Order Conditions, Necessary and Sufficient Conditions for Existence of Global Optima.
   (b) Equality Constrained Maximization: The Langrangean Setup and First and Second Order Conditions.
   (c) Some Economic Applications: Utility Maximization, Profit Maximization, and Utility Maximization with Labour/Leisure Choice.

Section II
Linear Algebra

3) Vector Spaces: Euclidean Spaces, Vector Algebra, Basis and Span, Inner Product and Norms of Vectors, Orthogonality.
4) **Finite Dimensional Subspaces**: Dimensions of Subspaces attached to a Matrix, Row and Null Space, Fundamental Theorem of Linear Algebra.

5) **Spectral Theory**: Eigenvalues and Eigen Vectors, Diagonalization and Decomposition of Matrices, Definiteness of Quadratic Forms.

Section III
Real Analysis

1) **Sequences in \( \mathbb{R}^n \)**: Definitions, Limits of Sequences, Convergent and Cauchy Sequences.

2) **Point-set Topology**: Open Sets, Closed Sets, Alternate definitions of Compactness, Heine-Borel Theorem, The Weierstrass Theorem.

Section IV
Static Optimization

1) **Functions and Calculus of Several Variables**: Functions between Euclidean Spaces, Total Derivative, Directional Derivative, the Gradient Vector and Hessian Matrix, The Implicit Function Theorem.

2) **Unconstrained Optimization**

3) **Constrained Optimization**: Equality and Inequality Constraints, The Kuhn-Tucker Formulation.

Section V
Dynamics

1) **First Order Linear Differential Equations** (Autonomous and Non-Autonomous), General and Specific Solutions.

2) **Two Dimensional Systems of First Order Differential Equations**.

3) **Linearization of Non-linear (Autonomous) Differential Equations/Systems**, Steady State and Stability Analysis, Phase Diagrams (Subject to availability of time).

Readings:
Core Texts:
**Sundaram, R.** : A First Course in Optimization Theory, Cambridge University Press.
**Simon, C.P. and Blume, L.** : Mathematics for Economists, Viva Norton India Edition

Additional Readings: