1. A glass is half-filled with milk and the other is half-filled with water. Both glasses are identical in shape, size and volume. From the first glass a teaspoon-full of milk is taken out and poured into the second glass. A teaspoon-full of the mixture in the second glass is then transferred to the first glass. After these two transfers:

   (a) More milk is transferred from glass 1 to glass 2

   (b) The amount of milk transferred from glass 1 is the same as the amount of water transferred from glass 2

   (c) Nothing can be said about the amounts of milk and water transferred

   (d) More water is transferred from glass 2 than milk from glass 1

2. The following steps lead to a fallacy:

   (1) $(-1)^2 = 1$

   (2) $\therefore 2\log(-1) = \log(1) = 0$

   (3) $\therefore \log(-1) = 0$

   (4) $\therefore (-1) = e^0 = 1$. That is, $-1 = 1$

   Which is the first incorrect step in the argument?

   (a) Step 1

   (b) Step 2

   (c) Step 3

   (d) Step 4

3. A car license plate has 6 digits. A superstitious owner is worried about the number "3" appearing anywhere in the license number. after some calculations he worked out the probability of getting an unlucky number. The correct value of this probability (to 2 decimal places) is:

   (a) 0.11

   (b) 0.17

   (c) 0.53

   (d) None of the above

4. "Every author really wants to have letters printed in the papers. Unable to make the grade, he drops down a rung of the ladder and writes novels."

   Which of the following best captures what is written above:

   (a) Authors stand on a ladder when writing

   (b) Letters in papers are better than novels

   (c) All novelists would have preferred to have written letters for newspapers

   (d) There are no good novelists

5. Select the word that is most similar in meaning to the word: ***[RUDE]***

   (a) Rough

   (b) Impolite

   (c) Uncouth

   (d) Protected

6. The value of $f(0)$, so that the function $f(x) = \frac{\sqrt{a^2-ax+x^2}-\sqrt{a^2+ax+x^2}}{\sqrt{a+x}-\sqrt{a-x}}$ becomes continuous for all x, given by:

   (a) $a^{\frac{3}{2}}$

   (b) $a^{\frac{1}{2}}$

   (c) $-a^{\frac{1}{2}}$

   (d) $-a^{\frac{3}{2}}$

7. Let $f(x) = [x]$ and $g(x) = \begin{cases} 0 & \text{if } x \in Z; \\ x^2 & \text{if } x \in R - Z. \end{cases}$, where R, Z and $\circ$ stands for the set of real numbers, integers and composition of functions respectively, then:

   (a) $g(x)$ is continuous at $x = 1$

   (b) $f(x)$ is continuous at $x = 1$

   (c) $g \circ f$ is continuous for all x

   (d) $f \circ g$ is continuous for all x

8. If $\log_{\frac{1}{3}} |z+1| > \log_{\frac{1}{3}} |z-1|$, then:
   (a) $Real(z) \geq 0$
   (b) $Real(z) < 0$
   (c) $Imaginary(z) > 0$
   (d) none of these

9. $C[0,1]$ is a set of continuous function with domain [0,1]. Then $(C[0,1], d)$ is a metric space under the metric d equal to:
   (a) $|\int_0^1 f(x) - g(x)dx|$
   (b) $\int_0^1 (f(x) - g(x))dx$
   (c) $|\int_0^1 (|f(x)| - |g(x)|)dx|$
   (d) none of these

10. Solve the equation:

$$\frac{dy}{dx} = \frac{3x^2 + 4x + 2}{2(y-1)} \qquad y(0) = -1$$

   (a) $y = -1 - \sqrt{2x^2 + 4}$
   (b) $y = 1 + \sqrt{x^3 + 2x^2 + 2x + 4}$
   (c) $y = 1 - \sqrt{2x^2 + 2x + 4}$
   (d) None of the above

11. The straight lines $x + y = 0$, $3x + y - 4 = 0$. and $x + 3y - 4 = 0$ form:
   (a) A right-angled triangle
   (b) An isoceles triangle
   (c) An equilateral triangle
   (d) None of the above

12. Given the following statements:

    S1: $\forall x(Baby(x) \rightarrow \neg Logical(x))$

    S2: $\forall x(ManageCrocodile(x) \rightarrow \neg Despised(x))$

    S3: $\forall x(\neg Logical(x) \rightarrow Despised(x))$

    Which of the following follows from S1–S3:

    (a) Babies cannot manage crocodiles

    (b) Crocodiles are despised

    (c) Nobody can manage crocodiles

    (d) Crocodiles are illogical

13. Suppose that f is a convex and differentiable function from the real line to the real line. Which of the following statements about its derivative f' is necessarily true?

    (a) f' is strictly increasing

    (b) f' is non-decreasing

    (c) f' is positive

    (d) None of the above

14. Suppose that $X_1, \ldots, X_n$ is an i.i.d. sample from Uniform $(0, \theta)$, where θ is an unknown parameter. Find out an unbiased estimator of θ based on

$$M := \max(X_1, \ldots, X_n)\,.$$

    (a) M

    (b) $nM/(n+1)$

    (c) $(n+1)M/n$

    (d) $2M$

15. 10 distinct balls are thrown one by one, at random into 20 distinct boxes. What is the expected number of empty boxes?

    (a) 10

    (b) 5

    (c) $19^{10}/20^9$

    (d) 0